more cumbersome algebra and integration. This is especially important when the expression for the bending moment consists of several terms. In the case of the cantilever beam with a single load the analysis can be performed just as easily with the aid of Eq. (6.7). The procedure in such a case is to make the direct substitution of $M_b = Wx$, squaring the term, and then integrating with respect to x.

Design Problem 7.2

Calculate the vertical displacement of the curved bar shown in Fig. 7.2, assuming constant rigidity and an elastic strain energy due to bending. Use Eq. (6.11) in the analysis.

Solution

The bending moment equation follows directly from Fig. 7.2:

$$M_b = WR\sin\theta \tag{7.2a}$$

The partial derivative of the moment with respect to the load W is

$$\frac{\partial M_b}{\partial W} = R\sin\theta \tag{7.2b}$$

Substituting the results, Eqs. (7.2a) and (7.2b), in Eq. (6.11) gives for the respective limits of integration of 0 and $\pi/2$

$$\begin{aligned} Y &= \int_0^{\pi/2} \frac{WR\sin\theta}{EI} \times R\sin\theta \times R\,d\theta \\ &= \frac{WR^3}{EI}\int_0^{\pi/2} \sin^2\theta\,d\theta \\ &= \frac{\pi WR^3}{4EI} \quad \blacklozenge \end{aligned} \tag{7.2c}$$

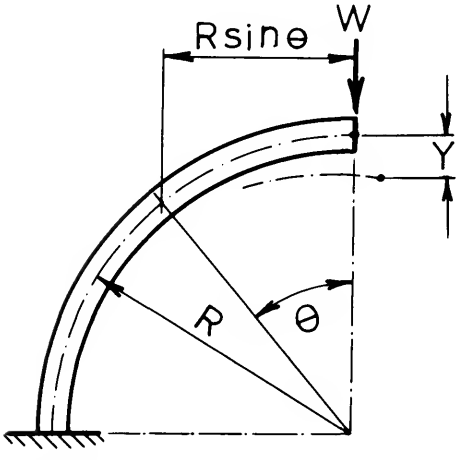

Fig. 7.2 Curved bar with end load.